# Cryptography

## Lecture 5: Pseudo One-Time Pad

November 12, 2024

# Overview

Contents

1. Classical cryptography
   (Shift & Vigenère cipher, one-time pad, perfect secrecy)
2. Security definitions & threat models
   (Computational security, CPA & CCA)
3. Private-key cryptography
   (Message authentication, hash functions, primitives, relevant ciphers)
4. Public-key cryptography
   (Assumptions, key management, digital signatures, relevant ciphers)

# Recall

## Definition (§4.2 Indistinguishability game)

Let $n \in \mathbb{N}$, $\mathcal{E} = (\mathrm{gen}, \mathrm{enc}, \mathrm{dec})$ efficient private-key encryption scheme and $\mathcal{A}$ stateful algorithm. Adversarial indistinguishability game $\mathrm{PrivK}^{\mathrm{one}}_{\mathcal{E},\mathcal{A}}(n)$

1. Security parameter $1^n$ sent to adversary $\mathcal{A}$
2. $\mathcal{A}$ selects messages $m_0, m_1 \in M$ with $|m_0| = |m_1|$
3. $k \leftarrow \mathrm{gen}(1^n)$; $b \leftarrow U_1$; $c \leftarrow \mathrm{enc}_k(m_b)$ sent to $\mathcal{A}$
4. $\mathcal{A}$ outputs bit $b' \in \{0,1\}$
5. Return 1 if $b = b'$ and 0 otherwise

## Definition (§4.3 EAV-secure)

Private-key encryption scheme $\mathcal{E} = (\mathrm{gen}, \mathrm{enc}, \mathrm{dec})$ is EAV-secure if for every PPT algorithm $\mathcal{A}$

$$\mathrm{P}\left[\mathrm{PrivK}^{\mathrm{one}}_{\mathcal{E},\mathcal{A}}(n)\right] \simeq \frac{1}{2}$$

# Recall

Alice

$k \leftarrow \text{gen}(1^n)$

random $b$

$c \leftarrow \text{enc}_k(m_b)$

Return $b \stackrel{?}{=} b'$

Security parameter $1^n$

Messages $m_0, m_1$ with $|m_0| = |m_1|$

Ciphertext $c$

Bit $b'$

Eve

Determines $b'$

# Recall

## Definition (§4.4 Pseudorandom generator)

Let $p$ be polynomial and $G$ deterministic polynomial-time algorithm such that $G(s) \in \{0,1\}^{p(n)}$ for all $s \in \{0,1\}^n$ and $n \in \mathbb{N}$.
$G$ is pseudorandom generator if

- $p(n) > n$ for all $n \geq 1$ (expansive)
- For any PPT algorithm $D\colon \{0,1\}^* \to \{0,1\}$

$$\sum_{s \in \{0,1\}^n} \frac{\mathrm{P}\big(D(G(s))\big)}{2^n} \simeq \sum_{r \in \{0,1\}^{p(n)}} \frac{\mathrm{P}\big(D(r)\big)}{2^{p(n)}}$$

# Pseudorandom Generator

## §4.5 Observation

If $P = NP$ then pseudorandom generators cannot exist

## Proof

Note that universal distinguisher of previous slide is in $NP$.
If $P = NP$, then it is polynomial-time and thus 2nd item of Definition §4.4 cannot be fulfilled. □

# Pseudorandom Generator

## §4.6 Definition

Given polynomial $p$, $p$-ensemble is function $\left(\mathfrak{X}_n \colon \{0,1\}^{p(n)} \to [0,1]\right)_{n\in\mathbb{N}}$ such that $\mathfrak{X}_n$ is probability distribution for all $n \in \mathbb{N}$.

$p$-ensembles $\mathfrak{X}$ and $\mathfrak{Y}$ are computationally indistinguishable, written $\mathfrak{X} \equiv_{\mathsf{PPT}} \mathfrak{Y}$, if for every PPT algorithm $D \colon \{1\}^* \times \{0,1\}^* \to \{0,1\}$

$$\mathrm{E}\Big[\mathrm{P}\big(D(1^n, x)\big)\Big]_{x\leftarrow\mathfrak{X}_n} \simeq \mathrm{E}\Big[\mathrm{P}\big(D(1^n, y)\big)\Big]_{y\leftarrow\mathfrak{Y}_n}$$

Notes

- Success probabilities of algorithms $x \leftarrow \mathfrak{X}_n \,;\, D(1^n, x)$ and $y \leftarrow \mathfrak{Y}_n \,;\, D(1^n, y)$ asymptotically almost equal
- 2nd item of §4.4 simplifies to $\left(\frac{|G^{-1}(r)|}{2^n}\right)_{r\in U_{p(n)}} \equiv_{\mathsf{PPT}} U_{p(n)}$

  (pseudorandom = computationally indistinguishable from uniform)

# Pseudorandom Generator

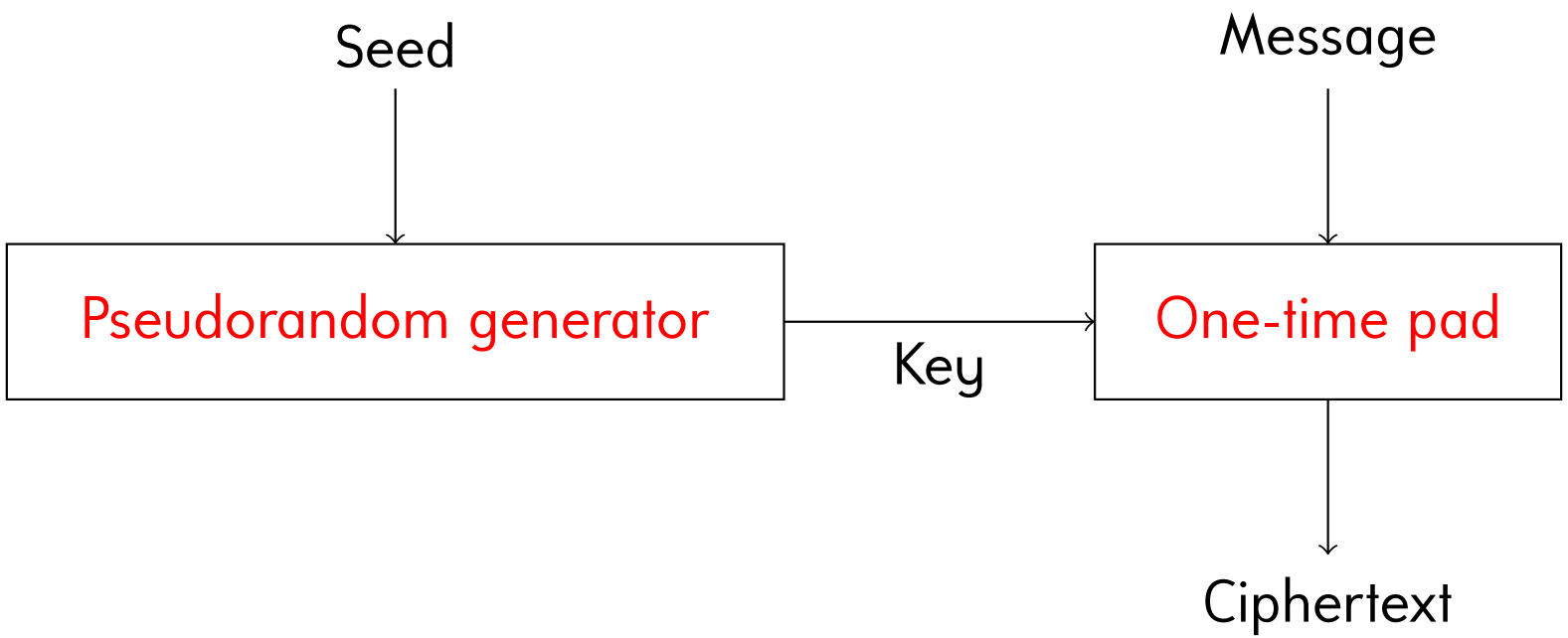
Questions

- Is $\equiv_{\mathsf{PPT}}$ equivalence relation?
- Does $\mathsf{P} \neq \mathsf{NP}$ imply existence of pseudorandom generator?
  (converse of Observation §4.5; worst- vs. average-case complexity)
  Careful: Would make cryptographers very happy if you prove this
- Are pseudorandom generators closed under composition?
  (Given pseudorandom generators $G_1$ and $G_2$
  is $G$ given by $G(s) = G_2(G_1(s))$ for all $s \in \{0,1\}^*$ again
  pseudorandom generator?)

# EAV-Secure Cipher

## Motivation

- One-time pad needs long random keys
- Pseudorandom generator can turn short random key (seed) into long pseudorandom key for one-time pad
  (that is computationally indistinguishable from random)
- Combination may solve long key problem



# EAV-Secure Cipher

## §4.7 Construction

Let $G$ be pseudorandom generator with expansion $p$.
Define private-key encryption scheme $\mathcal{E}_G = (\text{gen}, \text{enc}, \text{dec})$ with

- $\mathrm{P}_k^n(k) = 2^{-n}$ for all $n \in \mathbb{N}$ and $k \in \{0,1\}^n$
- $\text{enc}_k(m) = G(k) \oplus m$ for all $k \in K$ and $m \in M$ with $|m| = p(|k|)$
- $\text{dec} = \text{enc}$

Notes

- Can only encrypt messages $m$ of particular lengths $|m| \in \text{ran}(p)$
- Keys can be much shorter than messages; i.e. $n \ll p(n)$

# Multiple Encryptions

## Summary

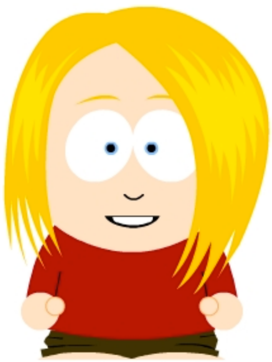
- One-time pad perfectly secret
- Pseudo one-time pad (Construction §4.7) EAV-secure
  but requires pseudorandom generator
- Security guarantee applies to key used once
- Both terrible in other threat models (in particular: key reuse)

## Famous "two-time pads"

- Venona project (1943–1980): Counterintelligence program by NSA
  (broke 3,000 messages due to reused key pages)
- MS-PPTP using MS-CHAP-v1 (all Windows versions since 98 and NT)
  (pseudo one-time pad with same seed for communication
  Client $\rightarrow$ Server & Server $\rightarrow$ Client)
- IEEE 802.11 WEP (1997): WLAN encryption standard
  (one-time pad keys repeat after $2^{24}$ frames $\approx$ 5h with high traffic)

# Multiple Encryptions

KRACK (Key Reinstallation Attack, 2017) against WPA2



Alice

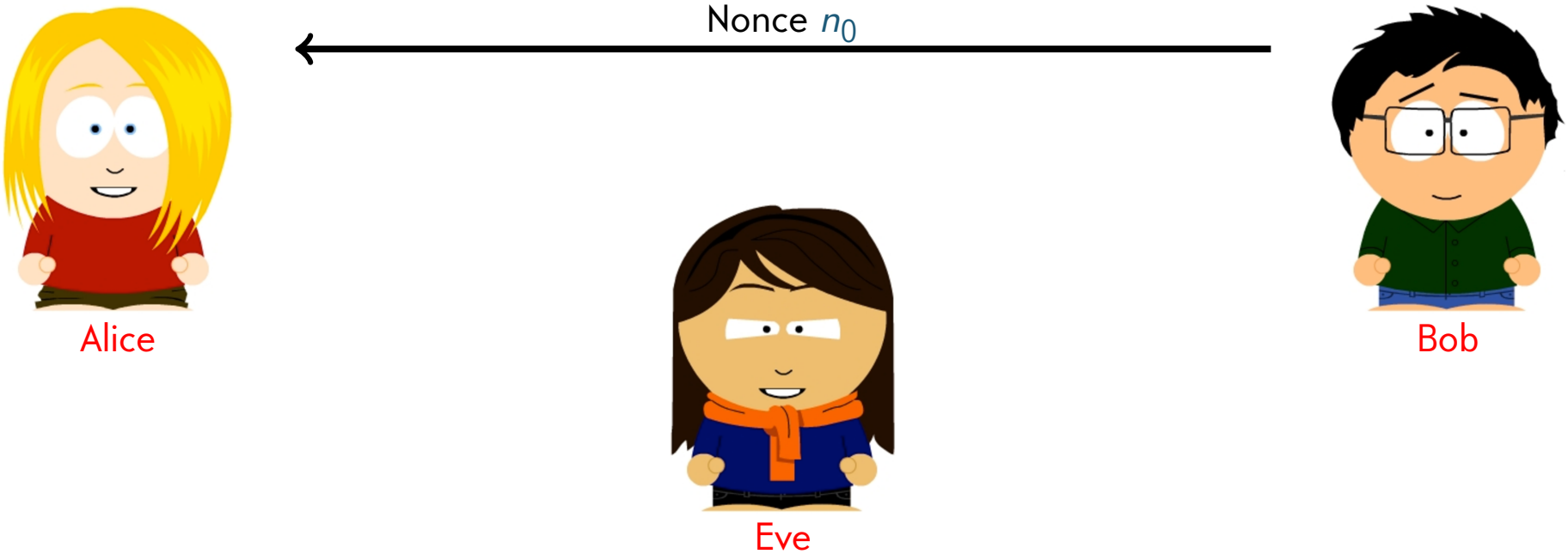
Eve

Bob

- Man-in-the-middle attack

# Multiple Encryptions

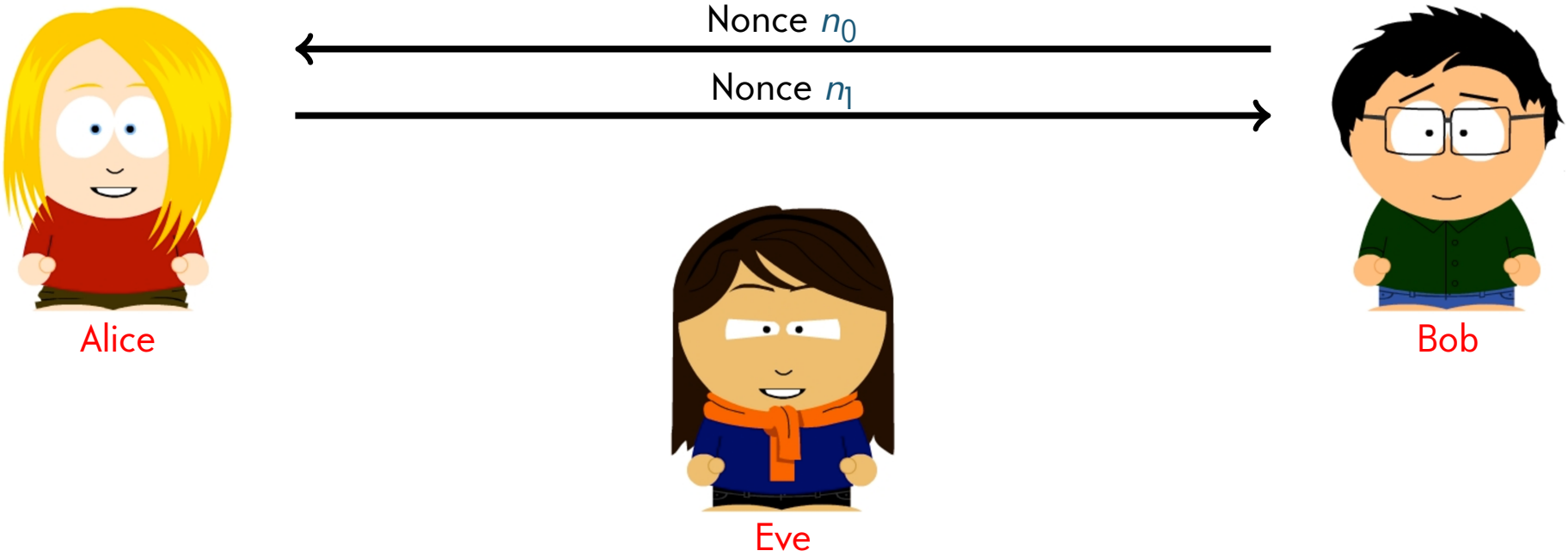
KRACK (Key Reinstallation Attack, 2017) against WPA2




- Man-in-the-middle attack

# Multiple Encryptions

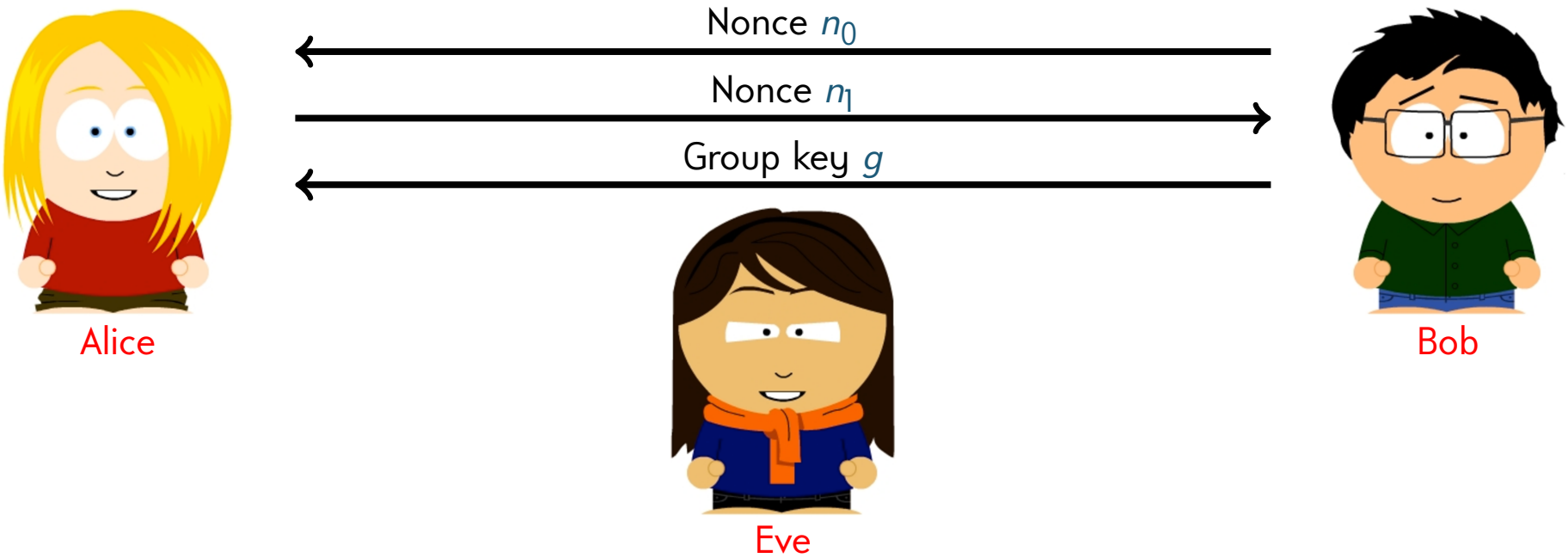
KRACK (Key Reinstallation Attack, 2017) against WPA2




- Man-in-the-middle attack

# Multiple Encryptions

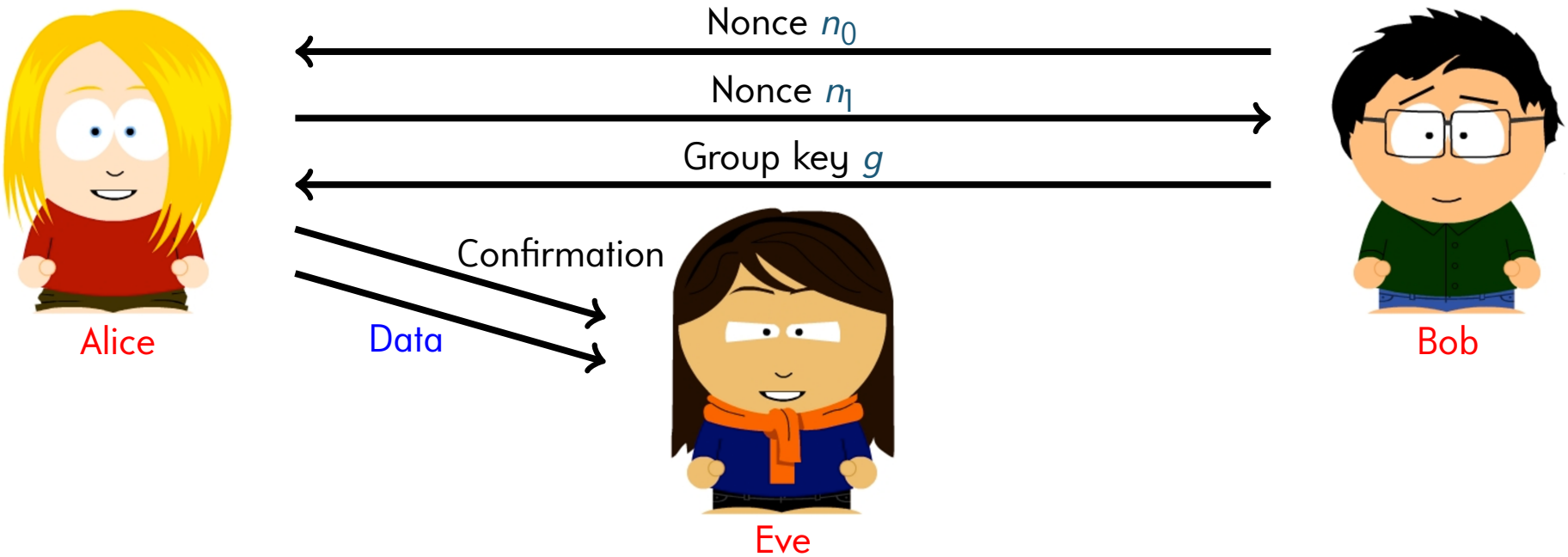
KRACK (Key Reinstallation Attack, 2017) against WPA2




- Man-in-the-middle attack
- After receiving $g$ Alice confirms & starts data exchange

# Multiple Encryptions

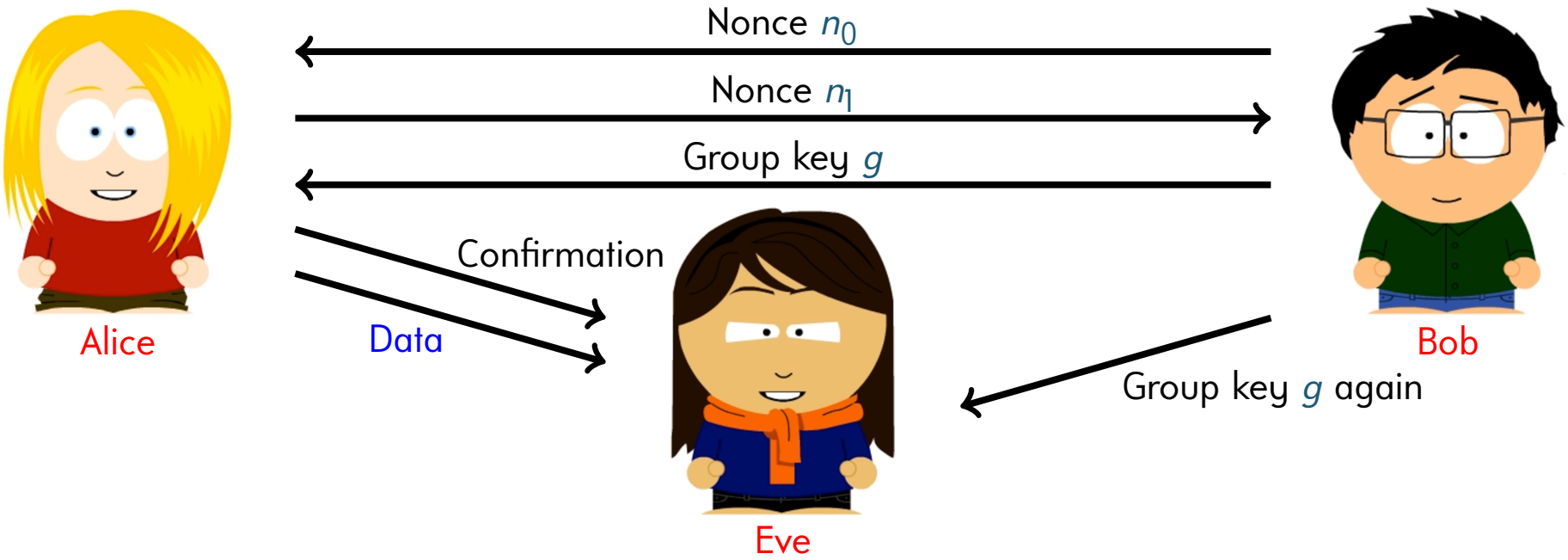
KRACK (Key Reinstallation Attack, 2017) against WPA2




- Man-in-the-middle attack
- After receiving $g$ Alice confirms & starts data exchange
- Eve intercepts confirmation which triggers Bob to resend $g$

# Multiple Encryptions

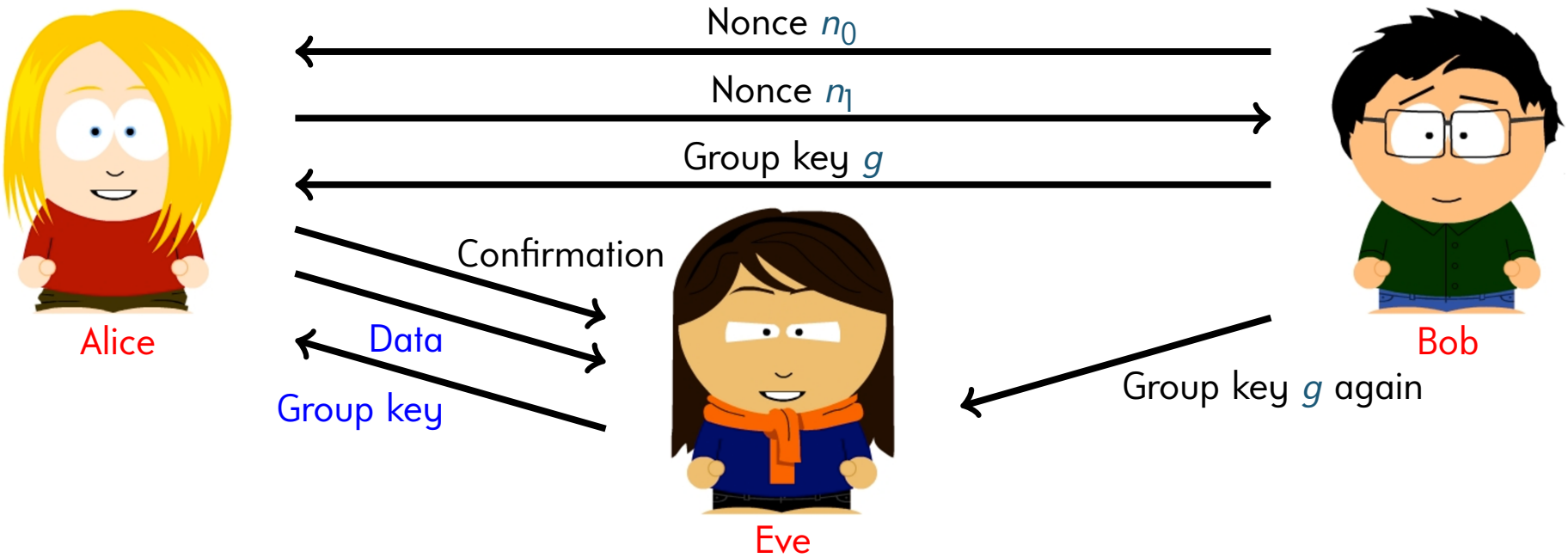
KRACK (Key Reinstallation Attack, 2017) against WPA2




- Man-in-the-middle attack
- After receiving $g$ Alice confirms & starts data exchange
- Eve intercepts confirmation which triggers Bob to resend $g$
- Eve delays resent $g$ until she has enough data

# Multiple Encryptions

KRACK (Key Reinstallation Attack, 2017) against WPA2




- Man-in-the-middle attack
- After receiving $g$ Alice confirms & starts data exchange
- Eve intercepts confirmation which triggers Bob to resend $g$
- Eve delays resent $g$ until she has enough data
- Resent $g$ causes Alice to reset & next data uses same key

# EAV-Secure Cipher

## §5.1 Theorem

Scheme $\mathcal{E}_G$ is EAV-secure for every pseudorandom generator $G$
(If $G$ is pseudorandom generator, then scheme $\mathcal{E}_G$ is EAV-secure)

## Proof (1/2)

By contraposition and reduction: Suppose that $\mathcal{E}_G$ is not EAV-secure, so there exists PPT algorithm $\mathcal{A}$ such that $\mathrm{P}\left[\mathrm{PrivK}^{\mathrm{one}}_{\mathcal{E}_G,\mathcal{A}}(n)\right] \not\approx \frac{1}{2}$.

We construct distinguisher $D$ with inputs $1^n$ and $w \in \{0,1\}^{p(n)}$ as follows:

1. Run $\mathcal{A}(1^n)$ to obtain messages $m_0, m_1 \in \{0,1\}^{p(n)}$
2. $b \leftarrow U_1$ and set $c = m_b \oplus w$
3. Let $b' \leftarrow \mathcal{A}(c)$ and return 1 if $b' = b$ and return 0 otherwise

Clearly $D$ is PPT algorithm since $\mathcal{A}$ is.

## Proof (2/2)

We claim that $D$ distinguishes $\left(\frac{|G^{-1}(r)|}{2^n}\right)_{r\in U_{p(n)}}$ from $U_{p(n)}$.

Let $\mathcal{E}_\ell$ denote one-time pad of length $\ell$. Algorithm $r \leftarrow U_{p(n)}$ ; $D(1^n, r)$ is essentially $\mathsf{PrivK}^{\mathsf{one}}(\mathcal{E}_{p(n)}, \mathcal{A})$, so by Theorems 2.9 and 3.3

$$\mathsf{E}[\mathsf{P}(D(1^n, r))]_{r\leftarrow U_{p(n)}} = \mathsf{P}[\mathsf{PrivK}^{\mathsf{one}}(\mathcal{E}_{p(n)}, \mathcal{A})] = \frac{1}{2}$$

On the other hand, algorithm $s \leftarrow U_n$ ; $r = G(s)$ ; $D(1^n, r)$ is $\mathsf{PrivK}^{\mathsf{one}}_{\mathcal{E}_G,\mathcal{A}}(n)$, so

$$\mathsf{E}[\mathsf{P}(D(1^n, G(s)))]_{s\leftarrow U_n} = \mathsf{P}[\mathsf{PrivK}^{\mathsf{one}}_{\mathcal{E}_G,\mathcal{A}}(n)] \not\approx \frac{1}{2} = \mathsf{E}[\mathsf{P}(D(1^n, r))]_{r\leftarrow U_{p(n)}}$$

Hence $\left(\frac{|G^{-1}(r)|}{2^n}\right)_{r\in U_{p(n)}} \not\equiv_{\mathsf{PPT}} U_{p(n)}$ and $G$ is no pseudorandom generator. □

# Stream Cipher

## Optimizations

- Encrypt shorter messages by discarding superfluous bits from pseudorandom generator
- Avoid generating those superfluous bits in stream cipher

## Stream cipher

- Generates bits one-by-one (or in larger blocks)
- Keeps state to remember current configuration
- Since pseudorandom generator is computable, there exists Turing machine (TM) that generates output sequentially (as in "Complexity theory" lecture)
  $\rightarrow$ Once output produced, stop TM & return (bit, configuration)
  $\rightarrow$ Next call is provided configuration & continues computation

# Stream Cipher

## §5.2 Definition (Stream cipher)

Stream cipher is pair $(\mathsf{init}, \mathsf{next})$ of deterministic algorithms

- $\mathsf{init}\colon \underbrace{\{0,1\}^*}_{\text{seed}} \times \underbrace{\{0,1\}^*}_{\text{initial vector}} \to \underbrace{\{0,1\}^*}_{\text{initial state}}$
  takes seed & initial vector; returns initial state
- $\mathsf{next}\colon \underbrace{\{0,1\}^*}_{\text{current state}} \to \underbrace{\{0,1\}^*}_{\text{output}} \times \underbrace{\{0,1\}^*}_{\text{next state}}$
  takes current state; returns output bits & next state

Stream cipher operation: Input seed $s$, initial vector $v$, rounds $\ell$

1. $s_0 = \mathsf{init}(s, v)$
2. for $i = 1$ to $\ell$: $\quad (b_i, s_i) = \mathsf{next}(s_{i-1})$
3. return $(b_1, \ldots, b_\ell)$

# Multiple Encryptions

Proposals of EAV-Security for Multiple Messages

# Multiple Encryptions

Alice

Eve

Notes

# Multiple Encryptions

Alice

Security parameter $1^n$

Eve

Notes

# Multiple Encryptions

Alice

Security parameter $1^n$

Messages $(m_0^1, \ldots, m_0^\ell)$ and $(m_1^1, \ldots, m_1^\ell)$ with $|m_0^i| = |m_1^i|$

Eve

## Notes

- Adversary provides two lists of messages

# Multiple Encryptions

Alice
$k \leftarrow \text{gen}(1^n)$
random $b$

Security parameter $1^n$

Messages $(m_0^1, \ldots, m_0^\ell)$ and $(m_1^1, \ldots, m_1^\ell)$ with $|m_0^i| = |m_1^i|$

Eve

## Notes

- Adversary provides two lists of messages

# Multiple Encryptions

Alice

$k \leftarrow \text{gen}(1^n)$

random $b$

$c_1 \leftarrow \text{enc}_k(m_b^1)$

$\ldots$

$c_\ell \leftarrow \text{enc}_k(m_b^\ell)$

Security parameter $1^n$

Messages $(m_0^1, \ldots, m_0^\ell)$ and $(m_1^1, \ldots, m_1^\ell)$ with $|m_0^i| = |m_1^i|$

Eve

## Notes

- Adversary provides two lists of messages
- Alice encrypts one of the lists & asks adversary which was encrypted

# Multiple Encryptions

Alice

$k \leftarrow \text{gen}(1^n)$

random $b$

$c_1 \leftarrow \text{enc}_k(m_b^1)$

$\ldots$

$c_\ell \leftarrow \text{enc}_k(m_b^\ell)$

Security parameter $1^n$

Messages $(m_0^1, \ldots, m_0^\ell)$ and $(m_1^1, \ldots, m_1^\ell)$ with $|m_0^i| = |m_1^i|$

Ciphertexts $(c_1, \ldots, c_\ell)$

Eve

## Notes

- Adversary provides two lists of messages
- Alice encrypts one of the lists & asks adversary which was encrypted

# Multiple Encryptions

Alice

$k \leftarrow \text{gen}(1^n)$

random $b$

$c_1 \leftarrow \text{enc}_k(m_b^1)$

$\ldots$

$c_\ell \leftarrow \text{enc}_k(m_b^\ell)$

Security parameter $1^n$

Messages $(m_0^1, \ldots, m_0^\ell)$ and $(m_1^1, \ldots, m_1^\ell)$ with $|m_0^i| = |m_1^i|$

Ciphertexts $(c_1, \ldots, c_\ell)$

Bit $b'$

Eve

Determines $b'$

## Notes

- Adversary provides two lists of messages
- Alice encrypts one of the lists & asks adversary which was encrypted

# Multiple Encryptions

Alice

$k \leftarrow \text{gen}(1^n)$

random $b$

$c_1 \leftarrow \text{enc}_k(m_b^1)$

$\ldots$

$c_\ell \leftarrow \text{enc}_k(m_b^\ell)$

Return $b \stackrel{?}{=} b'$

Security parameter $1^n$

Messages $(m_0^1, \ldots, m_0^\ell)$ and $(m_1^1, \ldots, m_1^\ell)$ with $|m_0^i| = |m_1^i|$

Ciphertexts $(c_1, \ldots, c_\ell)$

Bit $b'$

Eve

Determines $b'$

## Notes

- Adversary provides two lists of messages
- Alice encrypts one of the lists & asks adversary which was encrypted

# Multiple Encryptions

## §5.3 Definition (Multiple-encryption indistinguishability game)

Let $n \in \mathbb{N}$, $\mathcal{E} = (\mathrm{gen}, \mathrm{enc}, \mathrm{dec})$ efficient private-key encryption scheme and $\mathcal{A}$ stateful algorithm. Multiple-encryption adversarial indistinguishability game $\mathrm{PrivK}^{\mathrm{many}}_{\mathcal{E},\mathcal{A}}(n)$ is

1. Security parameter $1^n$ sent to adversary $\mathcal{A}$
2. $\mathcal{A}$ selects $\ell \in \mathbb{N}$, and message lists $(m_0^1, \ldots, m_0^\ell) \in \mathcal{M}^\ell$ and $(m_1^1, \ldots, m_1^\ell) \in \mathcal{M}^\ell$ with $|m_0^i| = |m_1^i|$ for all $1 \leq i \leq \ell$
3. $k \leftarrow \mathrm{gen}(1^n)$; $b \leftarrow U_1$; $c_1 \leftarrow \mathrm{enc}_k(m_b^1), \ldots, c_\ell \leftarrow \mathrm{enc}_k(m_b^\ell)$
   ciphertext list $(c_1, \ldots, c_\ell)$ sent to $\mathcal{A}$ (challenge ciphertexts)
4. $\mathcal{A}$ outputs bit $b' \in \{0,1\}$
5. Return 1 if $b = b'$ ($\mathcal{A}$ wins)
   Return 0 otherwise

# Multiple Encryptions

**§5.4 Definition (Multiple-encryption EAV-secure)**

Private-key encryption scheme $\mathcal{E} = (\mathsf{gen}, \mathsf{enc}, \mathsf{dec})$ is multiple-encryption EAV-secure if for every PPT algorithm $\mathcal{A}$

$$\mathsf{P}\left[\mathsf{PrivK}^{\mathsf{many}}_{\mathcal{E},\mathcal{A}}(n)\right] \simeq \frac{1}{2}$$

Notes

- Surprisingly strong requirement (as we will see)
- Rather irrelevant in practice

# Multiple Encryptions

Example: "Two-time pad"

- On receipt of $n$, adversary selects messages $(0^n, 0^n)$ & $(0^n, 1^n)$
- For any key $k$ Alice returns either

$$\underbrace{(0^n \oplus k, 0^n \oplus k)}_{(k,k)} \quad \text{or} \quad \underbrace{(0^n \oplus k, 1^n \oplus k)}_{(k,\bar{k})}$$

- Adversary receives $(c_1, c_2)$ and returns 0 if $c_1 = c_2$ and 1 otherwise
- Adversary always wins

# Summary

- Pseudo one-time pad
- Stream cipher
- Multiple-encryption EAV-secure